Twix Challenge - Chocolate Orange

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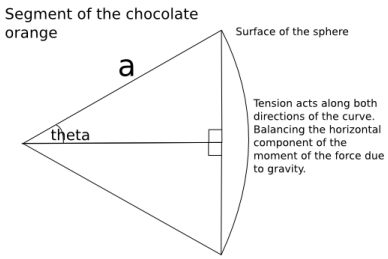
A chocolate orange of delicious, smooth, uniform chocolate of mass M, radius a, is sliced into segments by planes through a fixed axis. It stands horizontally on a table, with this axis vertical, and is held together by a narrow ribbon around its equator.

Show the tension in the ribbon is $\frac{3Mg}{32}$. You may assume that the centre of mass of a segment of angle 2θ is at distance $\frac{3\pi a sin\theta}{16\theta}$ from the axis.

By looking at the chocolate orange from above, and drawing a triangle of hypotenuse a, and resolving the force of tension so that it passes through the centre of the segment, we can see that the component of tension toward the axis for a segment of any value of θ is:

$$F_T = 2T sin\theta$$

Where T = *Tension*



The fraction of the sphere of chocolate that any segment is can be written as:

$\frac{2\theta}{2\pi} = \frac{\theta}{\pi}$

Therefore the mass represented by a segment is:

$$M = M_{total}\frac{\theta}{\pi}$$

Where M_{total} *is the mass of the whole chocolate orange.*

Therefore the force due to gravity is:

$$F_g = Mg\frac{\theta}{\pi}$$

The moment of a force is:

$$Moment = \overrightarrow{F} * \overrightarrow{r}$$

Where r represents the distance of the centre of mass from the point at which the chocolate orange touches the surface of the table. $\frac{3\pi a sin\theta}{16\theta}$

The sphere and its segments are stationary, so the force due to gravity and the tension provided by the ribbon must be equal.

$$Mg\frac{\theta}{\pi}\frac{3\pi a sin\theta}{16\theta} = 2aTsin\theta$$

Cancels to:

$$Mg\frac{3}{16} = 2T$$

Therefore:

$$T = Mg\frac{3}{32}$$